Contract-based Inter-user Usage Coordination in Free-floating Car Sharing

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Abstract

We propose a novel distributed user-car matching method based on a contract between users to mitigate the imbalance problem between vehicle distribution and demand in free-floating car sharing. Previous regulation methods involved an incentive system based on the predictions of origin-destination (OD) demand obtained from past usage history. However, the difficulty these methods have in obtaining accurate data limits their applicability. To overcome this drawback, we introduce contract-based coordination among drop-off and pick-up users in which an auction is conducted for drop-off users' intended drop-off locations. We theoretically analyze the proposed method regarding the upper bound of its efficiency. We also compare it with a baseline method and non-regulation scenario on a free-floating car-sharing simulator. The experimental results show that the proposed method achieves a higher social surplus than the existing method.

Introduction

This paper examines free-floating car sharing services. This service model allows users to travel one-way and to pick up and drop off cars anywhere in a dedicated service area. The high flexibility of such a usage model can be equivalent to that of private car ownership. At present, such services include SHARE NOW and Enjoy. Free-floating car sharing is an interesting application of multi-agent system technologies because it requires fine-grained coordination among drop-off and pick-up users.

Regardless of whether car sharing is free-floating or station-based, a central concern in car sharing studies is imbalance between the distributions of available vehicles and origin-destination (OD) demand, which ends up decreasing service efficiency. Many studies have tried to solve this problem by pricing usage (Jorge, Molnar, and Correia 2015; Febbraro, Sacco, and Saeednia 2012; Waserhole, Jost, and Brauner 2013; Singla et al. 2015; Reiss and Bogenberger 2017; Pan et al. 2019) and by predicting demand (Ciari, Balac, and Balmer 2015; Schmöller et al. 2015; Weikl and Bogenberger 2013; Li et al. 2019).

Free-floating service models make the problem of imbalance worse due to their flexibility (Weikl and Bogenberger 2013; Wielinski, Trépanier, and Morency 2015), i.e., cars are dropped off and picked up at many points. Existing studies often take the approach of mixed integer programming, but if we try to apply methods developed for station-based car sharing to free-floating car sharing, the increase in the number of pick-up/drop-off points makes solutions difficult to obtain. Some studies partition free-floating service areas into smaller zones that serve as virtual stations (Pan et al. 2019). This causes another problem of how to partition the service area so that pick-up/drop-off points can be efficiently repositioned. In addition, existing methods more or less assume that usage data is available for prediction. We may be able to collect such data, but it is difficult to accurately predict the demand and the price elasticity of demand at each point. This means that methods developed for obtaining solutions for station-based car sharing systems are difficult to extend to free-floating car sharing systems.

To solve this problem, we introduce negotiation among a pick-up user and drop-off users. A pick-up user asks drop-off users to place the car closer to her current location, and drop-off users may thus change their initially intended drop-off locations. More precisely, a pick-up user bids a function that indicates the cost of picking up the vehicle. Drop-off users bid their functions that indicate the additional cost for changing their drop-off point. We also consider the social value of vehicle usage. The use of a vehicle can be viewed as vehicle relocation for other pick-up users. A trip to a remote area is often valuable for many users. We develop a method for estimating relocation values and use the Vickrey-Clarke-Groves (VCG) mechanism to determine pairs of pick-up and drop-off users and the monetary transfers.

The previous methods can be considered as centralized regulation in that the operator designs adequate incentives assuming that demand predictions are accurate, whereas our method can be viewed as distributed regulation in that the regulation is executed among users through auctions. To the best of our knowledge, our method is the first regulation strategy for free-floating car sharing systems that satisfies (1) scalability against an increase in the number of pick-up/drop-off points (the set of OD demand points of potential pick-up/drop-off users), (2) no requirement to partition the service area, and (3) no requirement for accurate demand predictions. Also, by virtue of the VCG mechanism, misreporting of the cost functions and the destinations of drop-off users can be prevented.

Here, a question is to what extent can such distributed regulation improve the quality of services and social surplus. To examine this, we model pick-up and drop-off users on the basis of the existing studies and exhaustively run simulations. Our contribution can be summarized as follows.

- We propose a novel regulation method for free-floating car sharing that uses the VCG mechanism, which enables a pick-up user and drop-off users to negotiate drop-off/pick-up locations.
- We clarify the effectiveness of our regulation method and its characteristics for different demand structures in service areas on the basis of the results attained from simulation experiments. More specifically, we compare our method with the method proposed by (Singla et al. 2015). The results of a simulation experiment showed that our method outperforms the baseline method in terms of service levels and social surplus.

Related Work

Regulation methods have been discussed more for the imbalance problems in one-way car sharing than those in free-floating car sharing due to service history (Jorge, Correia, and Barnhart 2014; Febbraro, Sacco, and Saeednia 2012; Waserhole, Jost, and Brauner 2013; Singla et al. 2015; Reiss and Bogenberger 2017; Pan et al. 2019). The drawback of the previous regulation methods is that they explored adequate incentives for users on the basis of the predicted OD demand and price elasticity of demand obtained from previous usage history, so they are ineffective when demand predictions are inaccurate and usage history is sparse or unavailable. To overcome these problems, our proposed distributed regulation method does not assume that demand predictions are accurate to solve the imbalance problems between car distribution and demand in free-floating car sharing.

Some studies examined applying auctions to car sharing (Hara and Hato 2018; Angelopoulos et al. 2018). These studies determined the utilization plan in a rather centralized manner and did not consider negotiation between users, which is different from our study.

Another line of related work is analyses done to clarify user characteristics (Herrmann, Schulte, and Voß 2014; Leclerc, Trépanier, and Morency 2013). Their observations are used to set the simulation parameters in this study.

Problem Setting and Formulation

We formulate a problem with free-floating car sharing regulation. The car sharing system operates n cars in a dedicated service area. The types and conditions of the vehicles are all the same, and the users are indifferent to them.

We assume a road network consisting of nodes and links, and cars can be parked at any point on a road in the dedicated service area. If non-parking zones exist, the discussion below still holds.

The location l of a car corresponds to the coordinates of latitude and longitude in a continuous 2D space, $l \in \mathcal{L}$. We use a discrete time model, $t = 0, 1, 2, \cdots$. At $t = 0$, the status of the n vehicles is "driving" or "parked" at some locations. $\mathcal{D}_t$ and $\mathcal{P}_t$ represent the set of cars that are driving and parked at time t, respectively.

At each time from $t = 1$, at most one pick-up user a_t appears. We assume that if more than one pick-up users demand car usage at the same time, these requests are sequenced in an arbitrary order by reducing the time resolution. Pick-up user a_t is located at $l^o_{a_t}$ and wants to move to $l^d_{a_t}$. If the intended trip occurs, she obtain positive valuation v_{a_t}. Pick-up user a_t has disutility $-f_{a_t}(x)(\leq 0)$ depending on the time x required to pick up a vehicle by walking from $l^o_{a_t}$ to the location of the vehicle and/or waiting for the arrival of the drop-off user.

Users driving cars are drop-off users. $\mathcal{B}_t$ represents the set of drop-off users at time t. Drop-off user $b_j \in \mathcal{B}_t$ can drop off the vehicle at $l^{d'}_{b_j}$ different from his initially intended drop-off location $l^d_{b_j}$ and walk to $l^d_{b_j}$. In such a case, he has disutility $-g_{b_j}(x)(\leq 0)$ in accordance with the delay in arriving at the intended drop-off location $l^d_{b_j}$, i.e., the difference in time x between the new and original itineraries. Because the driving speed is faster than the walking speed, if he changes his itinerary, x is larger than zero. After pick-up user a_t starts using a vehicle, she changes her role to drop-off user b_j. We allow $f_{a_t}(x) \neq g_{b_j}(x)$. This is because the pick-up user may change her trip plan, e.g., she can use public transportation instead or cancel her trip before departure but cannot do so once she has started using the vehicle.

Pick-up user a_t may be able to negotiate with drop-off users and asks a drop-off user to park the car closer to $l^o_{a_t}$ by paying p_{a_t}. If the contract increases the utility for drop-off user b_j, he accepts the request, changes the drop-off location, and receives reward r_{b_j}. Here, we allow $p_{a_t} \neq r_{b_j}$. If $r_{b_j} > p_{a_t}$, the service operator covers the deficit of $r_{b_j} - p_{a_t}$. In congested areas, it might be difficult to find a parking slot. In such cases, the reservation of the parking slot needs to be incorporated into the negotiation. If the reservation fails, the negotiation also fails.

Pick-up user a_t may be able to use a parked car. We assume a virtual user b_0 when a_t uses a parked car and enters a contract with b_0. In this case, pick-up user a_t does not have to pay p_{a_t} because the car is not relocated. Virtual user b_0 does not receive reward r_{b_0}.

For pick-up user a_t, $c_k = (b_k, l_k)$ denotes the contract under which drop-off user b_k drops off his car at location l_k, which is different from the original drop-off location $l^d_{b_k}$. $c_0 = (b_0, l_{b_0})$ means a special contract under which pick-up user a_t uses the vehicle that is parked nearest to $l^o_{a_t}$, i.e., $b_0 = argmin_{i \in \mathcal{P}_t} |l^o_{a_t} - l_i|$. l_{b_0} is the parking location of b_0. $t_{a_t}(c_k)$ is a function that returns the time required to pick up a vehicle under contract c_k for pick-up user a_t. $t_{b_j}(c_k)$ is a function that returns the delay in arriving at the intended drop-off location $l^d_{b_j}$ for drop-off user b_j. Here, we assume that the users and the operators have accurate information on the trip duration between two locations either by walking or driving[1].

[1]Readers may be concerned about uncertainty. We assume that cars move at a constant speed, but delays might be caused due to

When users reach an agreement, the pick-up point for user a_t should be closer than the location of virtual user b_0. The set of candidate contracts can be denoted as follows.

$$\begin{aligned} C &= \{c_k\} \\ &= \{(b_k, l_k) | t_{a_t}((b_k, l_k)) \le t_{a_t}((b_0, l_{b_0}))\} \cup \{(b_0, l_{b_0})\} \end{aligned} \tag{1}$$

Here, $b_k \in \mathcal{B}_t$.

If contract c^* is entered into, the utilities for pick-up user a_t and drop-off user b_j are represented as follows.

$$u_{a_t} = v_{a_t} - f_{a_t}(t_{a_t}(c^*)) - p_{a_t} \tag{2}$$

$$u_{b_j} = -g_{b_j}(t_{b_j}(c^*)) + r_{b_j} \tag{3}$$

Once the contract is entered into, neither the pick-up user nor the drop-off user is allowed to break the contract. It may seem that we evaluate only the space proximity between a pick-up user and a drop-off user. However, the arguments in the utility functions are related to time. If a car is dropped off at a point near a pick-up user, but getting the car takes a long time, the pick-up user's utility would decrease, and she would not confirm such a contract. Thus, the time dimension of the problem is also included in the consideration.

Our objective is to find a contract that maximizes the social surplus, i.e., the sum of utilities under the assumption of individual rationality. Individual rationality means that neither the pick-up user nor drop-off user enters a contract under which they suffer loss. The budget limit of the operator is not set prior to providing the service.

Proposed Method

We propose a novel distributed regulation method for free-floating car sharing systems. The imbalance between vehicle distribution and demand will be reduced through user-to-user interaction by conducting a VCG auction, i.e., the adjustment of the drop-off/pick-up location. We assume the center holds information on current locations and intended drop-off locations for pick-up users.

An auction is conducted every time a pick-up user applies for usage. The following process (1-3) is executed.

1. User a_t applies for usage and reports the valuation of her trip, her disutility function, her current location ($l^o_{a_t}$), and intended drop-off location ($l^d_{a_t}$) to the center.
2. The center announces the auction to driving users $b_j \in \mathcal{B}_t$ and collects bids from them. These bids include information on their current location, their intended drop-off location, and their disutility function $g_{b_j}()$. User b_j is included in the auction if the constraint $t_{a_t}((b_j, l_j)) \le t_{a_t}((b_0, l_{b_0}))$ is satisfied.
3. Execute either of the two actions below.
   - If there is at least one eligible driving user: A VCG auction is conducted for the drop-off users' drop-off locations. Note that the pick-up user, as well as the eligible drop-off users, are bidders. The auction result is notified to participants, and the decision and monetary transfer are executed.
   - If there are no eligible driving users: the pick-up user chooses the action that maximizes her utility, either picking up a parked vehicle b_0 without an auction or cancelling the usage application.

In Step 2, we assume that drop-off users that already have won a bid do not participate in any rounds. That is, a pick-up user and drop-off users that have not already committed to a previous auction round are allowed to participate in the auction.

In Step 3, we use the VCG mechanism. Assuming road networks consisting of nodes and links, users and vehicles can be located at a node or a point on the links. We assume that a link consists of a finite number of points. By using the Dijkstra algorithm, we search for the point that maximizes the social surplus and determines the pickup/drop-off point l_k for each contract $c_k = (b_k, l_k)$.

A user's trip can be viewed as the relocation of a vehicle for other pick-up users, i.e., a user's trip affects the utilities of other pick-up users. We take such side effects into consideration. We define a location value $v_L(l)$ as the expected valuation at location l. Later, we specify the expression of $v_L(l)$. Also, we define a relocation value $v_R(l^o, l^d)$ for relocation from l^o to l^d. If the drop-off location of a_t is in an area with many usage requests, such relocation is valuable for many users. If the drop-off location is in an area with few usage requests, such relocation is less useful for many users. $v_R(l^o, l^d)$ is defined as follows.

$$v_R(l^o, l^d) = v_L(l^d) - v_L(l^o) \tag{4}$$

On the basis of declarations of the pick-up user and the driving users, contract c^* and the pick-up user's drop-off location l^* are selected so as to maximize the social surplus.

$$\begin{aligned} \{c^*, l^*\} = \mathop{\arg\max}_{c \in C, l \in \mathcal{L}} \big(& v_{a_t} - f_{a_t}(t_{a_t}(c)) - g_{b_j}(t_{b_j}(c)) \\ & + v_R(l^d_{b_j}, l) \big) \end{aligned} \tag{5}$$

, where a_t means the pick-up user, and b_j means the driving user included in contract c. If $l^* \neq l^d_{a_t}$, the center recommends that the pick-up user change its drop-off location to l^*. The amount of monetary transfer is calculated as follows, i.e., pick-up user a_t pays p_{a_t} to the center, and drop-off user b_j receives r_{b_j} from the center.

$$p_{a_t} = g_{b_j}(t_{b_j}(c^*)) - v_R(l^d_{b_j}, l^*) \tag{6}$$

$$\begin{aligned} r_{b_j} = -\Big(& \max_{c' \in C, c' \neq c^*, l \in \mathcal{L}} \big(v_{a_t} - f_{a_t}(t_{a_t}(c')) - g_{b'_j}(t_{b'_j}(c')) \\ & + v_R(l^d_{b'_j}, l) \big) - \big(v_{a_t} - f_{a_t}(t_{a_t}(c^*)) + v_R(l^d_{b_j}, l^*) \big) \Big) \end{aligned} \tag{7}$$

, where b'_j means the driving user included in contract c'. The center's deficit is equal to $p_{a_t} - r_{b_j}$.

After the auction, if a parked car is selected instead of driving users' cars, it is reserved for the pick-up user.

In estimating a location value $v_L(l)$, we assume that (1) the probability of finding pick-up users p_l at location l is

traffic jams. We suppose that the uncertainty can be dealt with by introducing a mechanism design with execution failure, which has already been studied (Porter et al. 2008).

given, (2) if a pick-up user at location l is allocated a vehicle, her destination is uniformly distributed in the service area, and (3) any pick-up users finish driving the car during a time slot.

Proposition 1. *Under assumptions (1), (2), and (3), $v_L(l)$ is given by the following function.*

$$v_L(l) = \frac{p_l}{1-(1-p_l)\beta}\Big(1+\frac{\beta}{1-\beta}p_{ave}\Big) \quad (8)$$

, where p_{ave} represents the probability of finding pick-up users averaged over all locations in the service area, and $\beta(0 \leq \beta < 1)$ is a discount factor for obtaining a value in the future.

Proof (sketch): Assume n time slots. If a usage request occurs at $t = 0$, the probability of finding pick-up users at $t = 1$ is p_{ave} because of assumption (2). If a usage request does not occur at $t = 0$, the probability of finding pick-up users at $t = 1$ is still p_l because the vehicle parked at location l does not move. By adding up the expected values for all cases of n-length occurrences, the expected valuation S_n for n time slots can be calculated as follows.

$$S_n = p_l \sum_{j=0}^{n-1} \beta^j Y_j \quad (9)$$

, where $Y_j = p_{ave} + (1-p_l)Y_{j-1}$, and $Y_0 = 1$. Because S_n is an expression of a power series, we can transform it into the following equation.

$$\begin{aligned} S_n = & \frac{p_l}{1-(1-p_l)\beta} \\ & \times \Big(1+\frac{\beta(1-\beta^{n-1})}{1-\beta}p_{ave} - (1-p_l)\beta^n \frac{p_{ave}}{p_l}\Big) \end{aligned} \quad (10)$$

By considering $n \to \infty$, we can get $v_L(l)$ as S_∞. □

Our method satisfies efficiency, individual rationality, and strategy-proofness under the assumption that $v_L()$ is correct.

Note that if a location value is not included in the calculation, our method does not require partitioning the service area, any historical data, nor demand predictions. If a location value is included in the calculation, our method requires somewhat partitioning the service area and historical data. However, it is different from the existing studies in the following points. First, in the existing studies, how the service area is partitioned is critical. In (Singla et al. 2015), a payment is controlled by the number of cars in an area, i.e., empty, saturated, or otherwise. These are discrete values. In comparison, in our method, a location value is continuous. Therefore, the way of partitioning does not change the outcome drastically. Second, a location value is updated after processing each usage request. This does not require a lot of historical data in advance.

Regarding the computational complexity, in this paper, we assume the center's availability, i.e., that the center announces the auction, collects the bids, and distributes the payments. However, it does not need to solve a large-scale aggregated optimization problem.

Analysis of Proposed Method

We show a theoretical analysis of the proposed method. Negotiation among a pick-up user and drop-off users enables drop-off/pick-up locations to be adjusted. However, it is not clear whether the accumulation of such local negotiations leads to global improvement. We show the upper bound of the improvement ratio in service levels. Here, service level means to what extent user requests are satisfied.

Apart from the discussion in the previous section, we distinguish whether a car is already parked or not in examining the probability of finding a pick-up user. Also, we do not consider a discount factor in this section. Assume that the car sharing system operates a single car. α denotes the probability that a pick-up user can be found after a drop-off user parks the vehicle at his intended location, i.e., the probability that a pick-up user exists within the range in which her utility of using the parked car is larger than or equal to zero. We assume that a usage request happens in a time slot and that the location of usage requests and the pick-up users' destinations are uniformly distributed in the service area. Also, any pick-up users finish driving the car during a time slot. In this case, in the succeeding n time slots, the expected number of vehicle utilizations is αn because a pick-up user's demand is independent from other users' demands. Thus, the service level is equal to $\alpha n/n = \alpha$.

We use $\Delta\alpha(> 0)$ to denote the increment of the probability that a pick-up user can be found when negotiation between the pick-up user and the drop-off user is introduced. Here, the following proposition holds.

Proposition 2. *The upper bound of the ratio of the service level in the proposed method to that in the non-regulation case is*

$$\Big(1+\frac{\Delta\alpha}{\alpha}\Big)(1+\Delta\alpha)^{n-1} \quad (11)$$

, where n represents the number of time slots.

Proof (sketch): When the proposed method is used, the probability of finding a pick-up user increases by $\Delta\alpha$. If we consider n time slots, vehicle utilization can be represented as a sequence of occupied or vacant time slots with a length of n. The probability of finding a pick-up user in time slot t is $\alpha + \Delta\alpha$ if the vehicle is used in time slot $t-1$; otherwise, it is α. Here, the expected number S_n of vehicle utilizations for n time slots can be calculated by adding up all possible cases, i.e., all n-length permutations of occupied and vacant.

$$\begin{aligned} S_n = & n(\alpha+\Delta\alpha)^n + (n-1)(\alpha+\Delta\alpha)^{n-1}(1-\alpha-\Delta\alpha) \\ & + (n-1)(\alpha+\Delta\alpha)^{n-2}(1-\alpha-\Delta\alpha)\alpha + \cdots \\ & + (1-\alpha-\Delta\alpha)(1-\alpha)^{n-2}\alpha \end{aligned} \quad (12)$$

When the number of occupied time slots is l out of n time slots, the probability that the vehicle is occupied for the first l time slots and vacant in the remaining $(n-l)$ time slots is larger than the probability of other cases including l occupied and $n-l$ vacant. Thus, we have the following.

$$
\begin{aligned}
S_n &\le n(\alpha + \Delta\alpha)^n \\
&\quad + \sum_{l=1}^{n-1} l \binom{n}{l} (\alpha + \Delta\alpha)^l (1 - \alpha - \Delta\alpha)(1 - \alpha)^{n-1-l} \\
&< n(\alpha + \Delta\alpha)^n \\
&\quad + \sum_{l=1}^{n-1} l \binom{n}{l} (\alpha + \Delta\alpha)^l (1 - \alpha)(1 - \alpha)^{n-1-l} \\
&= n(\alpha + \Delta\alpha)^n \\
&\quad + (\alpha + \Delta\alpha) \sum_{l=1}^{n-1} n \binom{n-1}{l-1} (\alpha + \Delta\alpha)^{l-1} (1 - \alpha)^{n-l} \\
&= n(\alpha + \Delta\alpha)((\alpha + \Delta\alpha) + (1 - \alpha))^{n-1} \\
&= n(\alpha + \Delta\alpha)(1 + \Delta\alpha)^{n-1} \qquad (13)
\end{aligned}
$$

By dividing S_n by n and α, we can obtain the ratio. □

This proposition indicates that the improvement ratio becomes large as the number of time slots increases if the increment $\Delta\alpha$ is small. Also, it indicates that the improvement becomes larger as the probability α becomes smaller. This is good news because we can avoid quite a low level of vehicle utilization by introducing our method.

Experiment Settings and Simulation Process

We constructed a free-floating car sharing simulator to evaluate the effectiveness and characteristics of our method. Our method was compared with an important previous regulation method and non-regulation scenario in the experiments. The regulation method by (Singla et al. 2015) was compared as a baseline with our proposal. Non-regulation scenario means that all drop-off drivers go to their intended drop-off points, and pick-up users use cars parked nearest their current locations if doing so offers a non-negative valuation.

Experiment Settings and Metrics

Experiment Settings. We configured the parameters of the free-floating car sharing simulator by referring to the literature (Herrmann, Schulte, and Voß 2014; Wielinski, Trépanier, and Morency 2015) and an existing free-floating car sharing service (Car2go). Table 1 shows the experiment settings. In OD demands, hubs represent points at which the spatial centralization of OD demands occurs. In 0 Hubs, there is no spatial centralization of OD demands. In 1 Hub, destination demands occur around the center of the service area for six hours, and origin demands occur around the center of the service area for the next six hours. This alternation is repeated. In 5 Hubs, there are five points at which the OD demands are centralized.

We referred to (Herrmann, Schulte, and Voß 2014) and constructed reasonable functions.

- Valuation function when picking up vehicle The valuation function of pick-up user a_t is composed of the positive valuation v_{a_t} obtained by the trip and the disutility obtained by the time x required to pick up a vehicle.

$$u_{a_t}(x) = v_{a_t} - 0.0001\alpha_{a_t} x^4 \qquad (14)$$

$$(v_{a_t}, \alpha_{a_t}) \in \{(6, 6), (5, 5), (4, 4), (3, 3), (2, 2), (1, 1)\}$$

This equation means that the user can get a parked car at most 10 mins away. One of the above six different parameter values will be assigned to each user in accordance with a uniform distribution. Here, we assume that pick-up users are concerned only about the walking/waiting time for vehicle usage, but we can incorporate other preferences such as vehicle preferences into the utility function.

- Valuation function when dropping off a vehicle On the basis of studies such as (Kato, Sakashita, and Tsuchiya 2015), we assumed that drop-off user b_j's disutility function can be formulated as follows.

$$u_{b_j}(x) = -0.21x \quad b_j \in \mathcal{B}_t \qquad (15)$$

Metrics. We used service levels, social surplus, and the center's deficit to evaluate the proposed method.

Service levels. Service levels are defined as (Potential customers - No-service events)/Potential customers, i.e., the ratio of the number of users who succeed in picking up a vehicle to the number of users who apply for usage.

Social surplus. The social surplus is defined as the total utilities of the users and the center obtained through the service process. It can be denoted as below.

$$SS = (V_p - W_p) - D_d \qquad (16)$$

We denoted the cumulative positive valuations obtained by the pick-up users for the trips as V_p, the cumulative disutilities obtained by the pick-up users for the time required to pick up vehicles $-W_p$, and the cumulative disutilities obtained by the drop-off users for the delay in reaching the initially intended destination $-D_d$. Note that relocation values were not included in the calculation of the social surplus to make the comparison possible.

Simulation Process

Simulation Process under Proposed Regulation. At the beginning of the process, all vehicles are parked in the service area in accordance with a uniform distribution. The iteration process finishes when the center takes a certain number of usage applications.

The center takes an application from pick-up user a_t with her current location $l^o_{a_t}$ at time t. Then, the vehicle distribution and the usage conditions are updated. If any driving users are in the service area, an auction is conducted. A contract is selected by the method described in the section Proposed Method, and the payment and reward are also calculated. If using a parked car maximizes her utility, she uses a parked car.

When applying our method, the probability of finding pick-up users p_l needs to be obtained. We divide the service area into an 80×160 grid and record usage requests in each sub-area. The usage requests in the past are discounted by β. We use the average value of the probability for 11×11 sub-areas centered on the location l as p_l.

The calculation of the negotiated drop-off location can be done in the following two steps. First, by assuming that the location is chosen from among the nodes in the road networks, we try to find the node that maximizes the social surplus. Then, we consider the rectangle in which these two

Service Area	A rectangular area 8 km in width and 16 km in length with 13,041 nodes and 25,840 links. To prevent the moving distances from having the same value for multiple routes, each node is fluctuated by a maximum of ± 20 m in the x and y coordinates.
Walking speed	80 m/min (4.8 km/h)
Driving speed	500 m/min (30 km/h)
Duration	For pick-up users, the larger value of (a) the distance between the pick-up and the current locations divided by the walking speed and (b) the distance between the negotiated drop-off location and the drop-off user's current location divided by the driving speed. For drop-off users, (i) the distance between the negotiated drop-off and current locations divided by the driving speed + (ii) the distance between the negotiated drop-off and initial drop-off locations divided by the walking speed - (iii) the distance between the initial drop-off and current locations divided by the driving speed.
Initial vehicle distribution	Vehicles are scattered uniformly at the beginning.
Number of vehicles	50, 200.
Demand frequency	Demand occurs in intervals of 30 s.
OD demands	0 Hubs, 5 Hubs, 1 Hub in the service area. If hubs exist in the service area, OD demand occurs in accordance with a multivariate normal distribution centered on the hubs with a standard deviation of 700 m. Whether the hubs become origins or destinations is altered every six hours.

Table 1: Experiment settings

nodes are diagonal points. Second, we try to find a location on the link that maximizes the social surplus. The node selected in Step 1 has at most four links. By moving from the node along each link, if the social surplus increases, we can find the location that maximizes the social surplus in a greedy manner because it has at most a single peak. If the social surplus decreases, we do not have to examine such links further. Thus, even if the number of candidate points increases, we can keep the cost of finding the point that maximizes the social surplus within a reasonable amount.

Simulation Process under Baseline. The method proposed by (Singla et al. 2015), DBP-UCB, mainly focuses on regulation for bike sharing, and the center gives monetary incentives to users to encourage them to pick up a vehicle at stations having a number of vehicles higher than a threshold and to drop off a vehicle at stations having a number of vehicles lower than the threshold. The amount of monetary incentives is determined by applying an extension of the budgeted multi-armed bandit algorithm.

In (Singla et al. 2015), the pick-up and drop-off locations are limited to stations designated prior to the service. To expand this regulation method to free-floating car sharing, the service area has to be divided into sub-areas that correspond to the stations assumed by the method. Therefore, we assumed that the service area (8000 m $\times$ 16,000 m) can be divided into an $8\times$ 8 grid in which each sub-area is a rectangle 1000-m wide and 2000-m long, considering that the distances between stations in a station-based sharing service are supposed to be worth using a vehicle to travel.

If we assume that OD demand is distributed uniformly in a service area, it is desirable for the vehicles to be distributed uniformly in the service area. This means that each sub-area should have 200/64 = 3.125 vehicles in the case of 200 vehicles and 64 sub-areas. Thus, we set the saturation criterion to 3. We used the same value in the cases of 5 Hubs and 1 Hub. In the case of 50 vehicles, the value was set to 1.

Under this regulation method, the center's deficit is the total cost spent as monetary incentives within the budget allocated in certain time periods.

Results and Discussion

We evaluated the service levels, the social surplus, and the center's deficit achieved by different regulation methods for different initial vehicle distributions under the same demand until the service had taken 10,000 usage applications.

Service Levels and Social Surplus Compared with Baseline

Table 2 shows the average service levels, the averaged social surplus, and the averaged center's deficit of 10,000 applications over 10 simulations for each different number of vehicles available under the different regulation methods. The standard deviation is shown in parentheses. It can be observed that even if the service level has similar values, the value of social surplus may differ significantly. Completing a usage request increases the service levels in the same manner, but the closer pick-up users can get a ride, the greater the social surplus.

Auction without v_R means the proposed method that ignores terms of relocation values, and auction with v_R means the proposed method including relocation values. Our method achieved higher service levels in 5 of the 6 cases and a higher social surplus in all cases, compared with the baseline.

The baseline method improved the average service level of the non-regulation scenario, but the improvement was not much in the cases of 50 vehicles. One reason could be that the reward for repositioning with the baseline method does not depend on the extra travel distance but whether different sub-areas are crossed. This result does not necessarily indicate that the baseline method was not effective in the cases of 50 vehicles, but it shows the difficulty of how to partition a service area into sub-areas. When we calculated the location values of two neighboring locations, the areas used for

	0 Hubs		5 Hubs		1 Hub	
	50 vehicles	200 vehicles	50 vehicles	200 vehicles	50 vehicles	200 vehicles
No regulation	0.306 (0.0046)	0.741 (0.0051)	0.113 (0.0017)	0.353 (0.0032)	0.114 (0.0026)	0.354 (0.0018)
Baseline	0.311 (0.0054)	**0.745** (0.0077)	0.154 (0.0031)	0.535 (0.0081)	0.157 (0.0026)	0.536 (0.0050)
Auction without v_R	0.382 (0.0035)	0.684 (0.0093)	0.220 (0.0342)	**0.589** (0.0312)	0.189 (0.0037)	**0.584** (0.0087)
Auction with v_R	**0.384** (0.0058)	0.613 (0.0147)	**0.254** (0.0517)	0.546 (0.0593)	**0.214** (0.0072)	0.544 (0.0180)
No regulation	7,276 (180)	20,178 (167)	3,041 (51)	10,223 (110)	3,077 (92)	10,357 (112)
Baseline	7,398 (146)	19,635 (175)	3,977 (75)	15,207 (264)	4,098 (116)	15,469 (128)
Auction without v_R	11,857 (182)	**22,649** (298)	8,193 (2,139)	**20,478** (2,304)	6,113 (146)	**20,060** (376)
Auction with v_R	**12,011** (224)	20,023 (478)	**9,496** (2,960)	18,900 (2,853)	**6,998** (292)	18,678 (697)
No regulation	0	0	0	0	0	0
Baseline	-106 (10)	-1,523 (47)	-238 (75)	-691 (38)	-237 (14)	-697 (31)
Auction without v_R	-4,568 (130)	-5,044 (159)	-3,033 (1,259)	-3,775 (1,250)	-1,603 (62)	-3,370 (99)
Auction with v_R	-4,681 (157)	-4,728 (149)	-3,416 (1,499)	-3,904 (1,105)	-1,900 (174)	-3,505 (115)

Table 2: Simulation results. Top: service levels, Middle: social surplus, Bottom: center's deficit. Standard deviation is shown in parentheses.

the calculation largely overlapped. Therefore, the location values of the neighboring locations are most likely to have similar values. However, Singla's method does not assume overlapping areas, which causes there to be a kind of on-off control. Whether an incentive is offered or not is sensitive to how the area is divided into sub-areas.

Table 2 (Bottom) shows that our method increased the center's deficit as well as the social surplus. The largest deficit seemed to be a limitation of our method. However, the deficit should be evaluated with the increase in the social surplus. The increase of social surplus from the non-regulation scenario was larger than the deficit amount in cases of 5 and 1 Hubs. In the experiments, a usage fee was not included in the evaluation because monetary transfer among users and the center does not affect the social surplus. If a for-profit company operates the system, the center can collect a usage fee from users. We can expect the amount of fees to be proportional to the social surplus, i.e., the satisfaction with the service. Although another experiment is needed for the detailed analysis, the deficit can be covered by the income of the usage fees. Thus, introducing our method into cases of high-demand concentrations is promising.

Effectiveness with Different Conditions

Number of vehicles available. When a small number of vehicles is dedicated for a service, our method is still effective. In such cases, there will be a low possibility that there are drivers whose intended drop-off locations are close enough to the pick-up users, i.e., $\Delta\alpha$ is close to zero. However, the probability of finding pick-up users, α, also becomes small. Thus, even if the number of dedicated vehicles is small, the improvement of our method is still large.

When the number of dedicated vehicles increases, the possibility that available vehicles are around the intended pick-up locations without an auction becomes high, i.e., α becomes large. Thus, if a large number of vehicles are available, the improvement of our method becomes small. These results coincide with the theoretical analysis.

Different demand distribution. We used three cases of 0 Hubs, 5 Hubs, and 1 Hub in the service area as a criterion for the degree of OD demand concentration. For 1 Hub, the OD demands were more concentrated than those for 5 Hubs. 0 Hubs corresponds to the case that the OD demands were extremely distributed. In all situations except for 0 Hubs with 200 vehicles, our method was highly effective. It is interesting to see that auction without v_R was better than auction with v_R in cases of 200 vehicles regarding the social surplus, while the former was worse than the latter in cases of 50 vehicles. When OD demand is concentrated, incorporating relocation values into auctions becomes effective in increasing the social surplus.

Execution performance. To deal with real-world situations, it seems sufficient to consider up to 1,000 vehicles. The simulation time ends in less than 100 seconds in the case of 1,000 vehicles with 1,000 usage requests (Intel Core i7-8565U 1.80 GHz, 16 GB of memory). Although the communication overhead needs to be considered in real-world situations, a contract can be determined in less than 0.1 seconds. It is a virtue that our method does not have to solve a large-scale optimization problem.

Conclusion

We proposed a novel distributed regulation method based on an auction between users to reduce the imbalance between vehicle distribution and demand in free-floating car sharing. Our method introduced the relocation value to take into account the impact of vehicle usage on subsequent users. In evaluations, we compared our method with the baseline and a non-regulation scenario in a free-floating car sharing simulator. The thorough evaluation showed that the proposed method achieved higher service levels in 5 of the 6 cases and a higher social surplus in all cases than the methods used for comparison. In addition, the effectiveness of the proposed method varied depending on the origin-destination (OD) demand concentrations and the number of dedicated vehicles. The proposed method is highly effective for severe demand concentrations, which is a critical situation among imbalance problems.

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